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GROUP PROPERTIES AND INVARIANT SOLUTIONS OF EQUATIONS DESCRIBING TWO-DIMENSIONAL FLOW OF GLACIERS

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One of the most important problems of contemporary glaciology is the construction of a mathematical theory of glacial mechanics, in which the development of mathematical models of glaciers plays a special role. Two different approaches can be distinguished in problems involving mathematical modeling of various processes. The first involves a tendency to construct a detailed model of the process under study, ensuring its adequacy by use of a large volume of experimental data, and then using the model to obtain quantitative conclusions and to apply such results in practice. The other approach involves construction of a spectrum of exact solutions for particular models, study of which would permit discovery of basic features of the process with less expenditure of time. Both directions are valid, and the results of the second can be used to justify and refine detailed mathematical models. The first approach was developed for glacial mechanics in [1-6], while the second has yet to enjoy such rapid growth. The results of the present study should be considered as a contribution toward the second approach toward mathematical modeling of glacial mechanics. In particular, the group properties of a nonlinear differential equation describing the position of the free surface of a glacier will be studied, invariant solutions of the equation will be constructed, and these solutions will then be used to study concrete problems arising in the study of glacier flow.

Considering the nonsteady state flow of a glacier in the isothermal approximation, it can be shown that the function $\ell(x, y, t)$ describing the free surface of the glacier satisfies a second-order nonlinear differential equation in partial derivatives

$$\begin{aligned}\frac{\partial l}{\partial t} = \frac{\partial}{\partial x}\left\{\left[\frac{\partial l}{\partial x}\Big/\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]\int\limits_{z_0}^{l}(l-z)\,\Gamma\left[(l-z)\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]dz\right\}+\\ +\frac{\partial}{\partial y}\left\{\left[\frac{\partial l}{\partial y}\Big/\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]\int\limits_{z_0}^{l}(l-z)\,\Gamma\left[(l-z)\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]dz\right\},\end{aligned} \tag{1}$$

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TABLE 1

Function z_0	Vectors ζ_i
Arbitrary	$\{\zeta_3\}$
$y^{\frac{\alpha+1-\lambda}{2\alpha+1}} f\left(\frac{x}{y}\right)$	$\zeta_3, \zeta_4, \zeta_5$
const	$\zeta_1, \zeta_2, \zeta_3, \zeta_4$
0	$\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5$

where t is time; x, y, spatial coordinates; $z_0(x, y)$, profile of the channel beneath the ice; $\Gamma(z)$, function characterizing the rheological properties of the ice, taken as a power function $\Gamma(z) = kz^{\alpha}$.

Commencing from Eq. (1) and corresponding boundary conditions for ℓ, all other characteristics of the glacier flow can be determined, in particular velocity in any direction, stresses produced within the glacier, etc. Thus, the basic problem of glacier-flow theory is determination of the form of the free surface ℓ.

Equation (1) is significantly nonlinear, and its solution can be obtained in general form only by approximate numerical methods. The absence of a priori estimates of the accuracy of numerical methods for solving equations of the form of Eq. (1) makes it necessary to construct analytical solutions, even if only for special self-similar cases, so that the corresponding difference methods can be tested. Some of these solutions, obtained on the basis of group analysis of Eq. (1), are of independent theoretical interest.

Construction of the full spectrum of invariant solutions of a concrete differential equation is based on the group properties of the equation [7]. We will consider the case in which $u = \ell - z_0$, assuming that $z_0(x, y)$ is an arbitrary function of its arguments, with $\alpha \geq 1$. The results of a group analysis of Eq. (1) in terms of tangent vector fields ζ_i of local single-parameter groups G_1 admitted by this equation which define the bases of Lie algebras of the corresponding infinitesimal operators are presented in Table 1, where $\{\zeta_3\}$ correspond to the base of the fundamental Lie algebras; the directional vectors ζ_i of the base infinitesimal operators X_i on which the nucleus is expanded depending on the concretization of the function $z_0(x, y)$ have the form

$$\zeta_1 = (1, 0, 0, 0, 0, 0),\ \zeta_2 = (0, 1, 0, 0, 0, 0),\ \zeta_3 = (0, 0, 1, 0, 0, 0),$$
$$\zeta_4 = (0,\ 0,\ t,\ -u/(1+2\alpha),\ -2v/(1+2\alpha),\ -2w/(1+2\alpha)),$$
$$\zeta_5 = (x,\ y,\ 0,\ u(1+\alpha)/(1+2\alpha),\ v/(1+2\alpha),\ w/(1+2\alpha));$$

the infinitesimal operators $X_i = \zeta_i\partial$. Here $\partial = (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial t,\ \partial/\partial u,\ \partial/\partial v,\ \partial/\partial w)$; v and w are auxiliary functions:

$$v = u\left(\partial z_0/\partial x + \partial u/\partial x\right)\left[1 + (w/v)^{2\alpha}\right]^{\frac{\alpha-1}{2\alpha}},$$
$$w = u\left(\partial z_0/\partial y + \partial u/\partial y\right)\left[1 + (v/w)^{2\alpha}\right]^{\frac{\alpha-1}{2\alpha}}.$$

We will use the data of Table 1 to construct invariant solutions of Eq. (1). To find the corresponding optimal systems of invariant solutions it is necessary to define all classes of such one- and two-dimensional subalgebras and their invariants. These are presented in Table 2. The most general and nontrivial of the second-range invariant solutions is the solution $\langle\lambda X_4 + X_5\rangle$, which is dependent on two arbitrary parameters α and λ. As follows from Table 2, this solution can be sought in the form $I_3 = \varphi(I_1, I_2)$ or

$$u(r,\ t) = t^{\frac{1+\alpha-\lambda}{\lambda(1+2\alpha)}}\psi(\eta),\ \eta = r^{\lambda}/t,\ r = \sqrt{x^2 + y^2}. \quad (2)$$

TABLE 2

Optimal subalgebras	Invariants	Invariant solutions	Channel form
$\langle\lambda X_4 + X_5\rangle$ $\forall\lambda$	$I_1 = \frac{x}{y}$, $I_2 = \frac{y^\lambda}{t}$, $I_3 = \frac{t}{u^{\frac{\lambda(1+2\alpha)}{1+\alpha-\lambda}}}$, $I_4 = \frac{u^{\frac{1-2\lambda}{1+\alpha-\lambda}}}{v}$, $I_5 = \frac{v}{w}$	second range: $u = t^{\frac{1+\alpha-\lambda}{\lambda(1+2\alpha)}}\varphi(I_1, I_2)$; $u = t^{\frac{1+\alpha-\lambda}{\lambda(1+2\alpha)}}\psi(\eta)$ $(\eta = r^\lambda/t)$	$z_0 = C + y^{\frac{1+\alpha-\lambda}{1+2\alpha}} f(I_1)$; $z_0 = Cr^{\frac{1+\alpha-\lambda}{1+2\alpha}}$ $(C = \text{const})$
$\langle\sin\beta\, X_1 + \cos\beta\, X_2\rangle$ $\forall\beta$	$I_1 = x\cos\beta - y\sin\beta$, $I_2 = t$, $I_3 = u$, $I_4 = v$, $I_5 = w$	$u = \varphi(I_1, I_2)$	$z_0 = \text{const}$
$\langle X_3\rangle$	$I_1 = x$, $I_2 = y$, $I_3 = u$, $I_4 = v$, $I_5 = w$	$u = \varphi(I_1, I_2)$	z_0 — Arbitrary function
$\langle\sin\beta\, X_1 + \cos\beta\, X_2 + \gamma X_4\rangle$ $\forall\beta,\ \gamma \neq 0$	$I_1 = x\cos\beta - y\sin\beta$, $I_2 = \gamma y - \cos\beta\cdot\ln t$, $I_3 = tu^{1+2\alpha}$, $I_4 = \frac{u^2}{v}$, $I_5 = \frac{v}{w}$	$u = t^{-\frac{1}{1+2\alpha}}\varphi(I_1, I_2)$	$z_0 = \text{const}$
$\langle\lambda X_4 + X_5,\ \sin\beta X_1 + \cos\beta\, X_2\rangle$ $\forall\lambda,\ \beta$	$I_1 = \frac{(y\sin\beta - x\cos\beta)^\lambda}{t}$, $I_2 = \frac{t}{u^{\frac{\lambda(1+2\alpha)}{1+\alpha-\lambda}}}$, $I_3 = \frac{u^{\frac{1-2\lambda}{1+\alpha-\lambda}}}{v}$, $I_4 = \frac{v}{w}$	first range: $u = t^{\frac{1+\alpha-\lambda}{\lambda(1+2\alpha)}}\varphi(I_1)$	$z_0 = \text{const}$
$\langle\lambda X_4 + X_5, X_3\rangle$ $\forall\lambda$ $\forall\beta$	$I_1 = \frac{x}{y}$, $I_2 = \frac{y^\lambda}{u^{\frac{\lambda(1+2\alpha)}{1+\alpha-\lambda}}}$, $I_3 = \frac{u^{\frac{1-2\lambda}{1+\alpha-\lambda}}}{v}$, $I_4 = \frac{v}{w}$	$u = y^{\frac{1+\alpha-\lambda}{1+2\alpha}}\varphi(I_1)$	$z_0 = C + y^{\frac{1+\alpha-\lambda}{1+2\alpha}} f(I_1)$, $C = \text{const}$
$\langle\sin\beta\, X_1 + \cos\beta\, X_2, X_3\rangle$ $\forall\beta$	$I_1 = x\cos\beta - y\sin\beta$, $I_2 = u$, $I_3 = v$, $I_4 = w$	$u = \varphi(I_1)$	$z_0 = \text{const}$
$\langle\sin\beta\, X_1 + \cos\beta\, X_2,\ \sin\beta\, X_1 + \cos\beta\, X_2 + \gamma X_4\rangle$ $\forall\beta,\ \gamma \neq 0$	$I_1 = x\cos\beta - y\sin\beta$, $I_2 = tu^{1+2\alpha}$, $I_3 = \frac{u^2}{v}$, $I_4 = \frac{v}{w}$	$u = t^{-\frac{1}{1+2\alpha}}\varphi(I_1)$	$z_0 = \text{const}$
$\langle X_3,\ \sin\beta\, X_1 + \cos\beta\, X_2 + \gamma X_4\rangle$ $\forall\beta,\ \gamma \neq 0$	$I_1 = x\cos\beta - y\sin\beta$, $I_2 = \gamma y + \cos\beta\cdot(1 + 2\alpha)\cdot\ln u$, $I_3 = \frac{u^2}{v}$, $I_4 = \frac{v}{w}$	$u = e^{-\frac{\gamma y}{(1+2\alpha)\cos\beta}}\varphi(I_1)$	$z_0 = \text{const}$

Substituting Eq. (2) in Eq. (1), we arrive at an ordinary differential equation for the function $\psi(\eta)$:

$$\lambda^{\alpha+1}\eta^{\frac{\lambda-1}{\lambda}}\left(\eta^{\frac{(\lambda-1)\alpha}{\lambda}}\psi^{\alpha+2}\psi'\,|\,\psi'\,|^{\alpha-1}\right)' + \lambda^\alpha\eta^{\frac{\alpha(\lambda-1)-1}{\lambda}}\psi^{\alpha+2}\psi'\,|\,\psi'\,|^{\alpha-1} + \eta\psi' - \frac{1+\alpha-\lambda}{\lambda(2\alpha+1)}\psi = 0. \tag{3}$$

As solutions of the problem with moving "front" we must choose only those solutions of Eq. (3) which satisfy the boundary condition $\psi|_{\eta=\eta_0} = 0$ for $\eta_0 \neq 0$. Then, from Eq. (2),

$$r_0 = (\eta_0 t)^{1/\lambda}. \tag{4}$$

We will enumerate the concrete solutions which can be found in this manner.

Solution 1. Planolinear Spreading of the Glacier on a Horizontal Bed. Taking $\lambda = 3\alpha + 2$ in Eqs. (2)-(4), we obtain the analogous problem which was considered in [8].

Solution 2. Radial Spreading of the Glacier on a Horizontal Bed. At the initial moment let the entire ice mass be concentrated at the point $\eta = 0$, upon which spreading commences. In this case we add to Eq. (3) the conditions

$$u|_{t=0} = \delta(\eta),\ \psi|_{\eta=\eta_0} = Q|_{\eta=\eta_0} = Q|_{\eta=0} = 0,\ \int_0^{\eta_0} \eta\psi(\eta)\, d\eta = 1,$$

where $\delta(\eta)$ is a Dirac function; $Q = \psi^{\alpha+2}\psi'|\psi'|^{\alpha-1}$ is the ice mass loss at the boundaries of the glacier. The latter of these (condition of constancy of mass) gives $\lambda = 5\alpha + 3$. The function $\psi(\eta)$ and constant η_0 are defined from Eq. (3) with consideration of boundary conditions. Finally, we find

$$u(r,\ t) = t^{-\frac{2}{5\alpha+3}} \left[\frac{2\alpha+1}{(5\alpha+3)^{1/\alpha}(\alpha+1)}\right]^{\frac{\alpha}{2\alpha+1}} \left(\eta_0^{\frac{\alpha+1}{\alpha(5\alpha+3)}} - \eta^{\frac{\alpha+1}{\alpha(5\alpha+3)}}\right)^{\frac{\alpha}{2\alpha+1}},$$

$$\eta = r^{5\alpha+3}/t,\ r_0 = (\eta_0 t)^{1/(5\alpha+3)},\ \eta_0 > 0.$$

For a glacier of unit volume

$$\eta_0 = \left\{\left[\frac{\alpha+1}{(2\alpha+1)(5\alpha+3)^2}\right]^{\frac{\alpha}{2\alpha+1}} \left[\frac{\alpha+1}{\alpha B\left(\frac{2\alpha(5\alpha+3)}{\alpha+1},\ \frac{3\alpha+1}{2\alpha+1}\right)}\right]\right\}^{\frac{(5\alpha+3)(2\alpha+1)}{20\alpha^2+23\alpha+7}}$$

[where B(x, y) is a beta function].

Let $l(r,\ t)$ be a solution. As was shown in the problem of [9] involving flow of a glacier with arbitrary initial distribution, at large values of t it will tend to u(r, t), i.e.,

$$l(r,\ t) = u(r,\ t) + o\left(t^{-\frac{2}{5\alpha+3}}\right).$$

Thus, the solution found not only gives a qualitative picture of glacier flow, it permits evaluation of glacier behavior at large time values.

Solution 3. Radial Spread of a Glacier in a Channel $z_0 = r^{-2}$. From Eqs. (2)-(4) at $\lambda = 5\alpha + 3$ and the ice mass budget

$$F = t^{-\frac{5(\alpha+1)}{5\alpha+3}} \eta^{\frac{5\alpha+1}{5\alpha+3}} f(\eta)$$

[where $f(\eta)$ is a function dependent on the invariant η], it follows that

$$u(r,\ t) = t^{-\frac{2}{5\alpha+3}} \left[\psi(\eta) + \eta^{-\frac{2}{5\alpha+3}}\right],\ \eta = r^{5\alpha+3}/t,$$

$$r_0 = (\eta_0 t)^{\frac{1}{5\alpha+3}},\ r_1 = (\eta_1 t)^{\frac{1}{5\alpha+3}},\ \eta_0 > 0.$$

The function $\psi(\eta)$ and constants η_0 and η_1 for specified f and α are defined from Eq. (3) with boundary conditions

$$\eta_0|_{t=0} = \eta_1|_{t=0} = \psi|_{\eta=\eta_0} = \psi|_{\eta=\eta_1} = Q|_{\eta=\eta_0} = Q|_{\eta=\eta_1} = 0,$$

$$Q = \psi^{\alpha+2}\psi'|\psi'|^{\alpha-1}.$$

This solution is of interest in that it allows prediction of the form of a glacier descending from a mountain slope with a specified ice mass budget.

Solution 4. Model of a Glacier with Fixed Boundary (planolinear flow). At the initial moment let the free surface be described by the equation

$$l|_{t=0} = \begin{cases} k(-x)^{\frac{\alpha+1}{2\alpha+1}}, & x \leqslant 0, \\ 0, & x > 0. \end{cases}$$

At $x = 0$, $\ell = 0$, $Q = 0$. We will seek a solution in the form $\langle X_3 + \lambda X_4 \rangle$ [8] at $\lambda = -2$, $z_0 = 0$; then

$$l(x, t) = (t_0 - t)^{-\frac{1}{2\alpha+1}} B(-x)^{\frac{\alpha+1}{2\alpha+1}},$$
$$t_0 = (B/k)^{2\alpha+1}, \quad B = \left[\frac{(2\alpha+1)(3\alpha+2)}{\alpha+1}\right]^{\frac{\alpha(4\alpha+3)}{(3\alpha+2)^2}} \Big/ (3\alpha+2)^{\frac{\alpha+1}{3\alpha+2}}. \tag{5}$$

Solution 5. Model of a Glacier with Fixed Boundary (radial flow). This model is similar to the preceding one. From Eqs. (2)-(4) at $z_0 = 0$, $\eta^{1/\lambda} = r/t^{1/\lambda}$, $\lambda \to \infty$ we find the exact solution for determining the free surface of the glacier

$$l(r, t) = (t_0 - t)^{-\frac{1}{2\alpha+1}} r^{\frac{\alpha+1}{2\alpha+1}} \left[\frac{2\alpha+1}{(\alpha+1)(5\alpha+3)^{1/\alpha}}\right]^{\frac{\alpha}{2\alpha+1}}. \tag{6}$$

Solutions (5) and (6) are interesting in that they permit prediction of the existence of a glacial form such that over the course of a certain time the edge of the glacier will remain immobile and will begin to move only after a corresponding readjustment of the profile.

Solution 6. Glacier Located in a Steady State on a Channel $z_0 = (r\cos\varphi)^{\frac{\alpha-1}{2(\alpha+1)}}\left(\varphi = \operatorname{arcctg}\frac{x}{y}, r = \sqrt{x^2 + y^2}\right)$. We turn again to Table 2. The most general and nontrivial of the first range invariant solutions is the solution $\langle \lambda X_4 + X_5, X_3 \rangle$, which depends on the two arbitrary parameters α and λ. As follows from Table 2, we may seek this solution in the form

$$u = y^{\frac{1+\alpha-\lambda}{1+2\alpha}} \varphi(\eta) = (r\sin\varphi)^{\frac{1+\alpha-\lambda}{1+2\alpha}} \psi(\eta),$$
$$\eta = \frac{x}{y} = \operatorname{ctg}\varphi, \quad r = \sqrt{x^2+y^2}. \tag{7}$$

Substituting Eq. (7) in Eq. (1) to determine the function $\psi(\eta)$ at $\lambda = (3 + 5\alpha)/[2(\alpha + 1)]$, we obtain an analog of Eq. (3):

$$\left\{\psi^{\alpha+2}\left[(\psi'+f')^2 + \left(\frac{\alpha-1}{2(\alpha+1)}(\psi+f) - \eta(\psi'+f')\right)^2\right]^{\frac{\alpha-1}{2}} \left[(\psi'+f') - \eta\left(\frac{\alpha-1}{2(\alpha+1)}(\psi+f) - \eta(\psi'+f')\right)\right]\right\}' = 0. \tag{8}$$

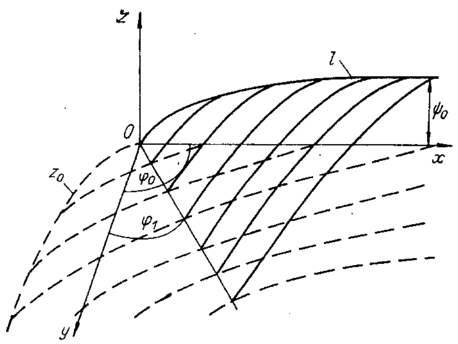

Fig. 1

Here $\psi(\eta)$ and $f(\eta)$ are functions dependent solely on the invariant η; $F = f(\eta)$ is the ice mass budget. The function $\psi(\eta)$ and constants η_0 and η_1 for specified ψ_0, $f(\eta)$ are defined from Eq. (8) with boundary conditions

$$\psi|_{\eta=\eta_1} = Q|_{\eta=\eta_0} = Q|_{\eta=\eta_1} = 0,$$

where

$$Q = \psi^{\alpha+2}\left[(\psi' + f')^2 + \left(\frac{\alpha-1}{2(\alpha+1)}(\psi+f) - \eta(\psi'+f')\right)^2\right]^{\frac{\alpha-1}{2}}\left[\eta\left(\frac{\alpha-1}{2(\alpha+1)}(\psi+f) - \eta(\psi'+f')\right) - (\psi'+f')\right].$$

The solution of the problem thus formulated is easily found in the closed form

$$l = r^{\frac{\alpha-1}{2(\alpha+1)}}\psi_0, \quad \psi_0 < 1,$$
$$f(\eta) = \eta^{\frac{\alpha-1}{2(\alpha+1)}},$$
$$\eta_0 = \operatorname{ctg}\varphi_0, \quad \eta_1 = \operatorname{ctg}\varphi_1,$$
$$\varphi_0 \leqslant \frac{\pi}{2}, \quad \varphi_1 = \arccos\left[\psi_0^{\frac{2(\alpha+1)}{\alpha-1}}\right].$$

The solid lines of Fig. 1 show glacier surfaces, with dashed lines being the glacier channels.

It is of interest to find the solution of significantly two-dimensional problems with a complex channel base. An attempt was made to construct such a solution for a channel form $z_0 = x^2/y^4 + \mathrm{const}/y^2$. However, such problems are not trivial, and there are definite difficulties found in solving them. Therefore, significantly two-dimensional problems with a complex channel require special attention.

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